RAMAKRISHNA MISSION VIDYAMANDIRA

(A Residential Autonomous College under University of Calcutta)

First Year

First-Semester Examination, December 2010

Date : 15-12-2010 **PHYSICS (Honours)** Full Marks : 75

Time : 11am – 2pm **Paper - I**

(Use separate answer script for each group)

Group – A

Answer any five questions : [10×5 = 50]

1. a) Find the eigenvalues and the eigen vector corresponding to the highest eigenvalue of the matrix

$$A = \begin{pmatrix} 5 & 0 & \sqrt{3} \\ 0 & 3 & 0 \\ \sqrt{3} & 0 & 3 \end{pmatrix}$$

b) Show that— i) $P_n(-x) = (-1)^n P_n(x)$ ii) $P_{2n}(0) = (-1)^n \frac{1.3.5...(2n-1)}{2^n n!}$ [5+2+3]

[Use generating function method]

2. a) An arbitrary vector $\vec{p}$ can be written as $\vec{p} = \alpha.\vec{A} + \beta\vec{B} + \gamma\vec{C}$, where $\alpha, \beta \ \& \gamma$ are constant coefficients and $\vec{A}, \vec{B} \ \& \ \vec{C}$ are non-coplanar vectors. How can $\alpha, \beta \ \& \gamma$ can be determined?

b) The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z^2$
A mosquito located at (1, 1, 2) desired to fly in such a direction that he will get cool as soon as possible. What direction should be chosen?

c) A Vector is given by $\vec{F} = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$, where symbols have usual meanings. Calculate $\vec{\nabla} \times \vec{F}$. [3+3+4]

3. a) Show that $\vec{F} = (2xy - z^2)\hat{i} + x^2\hat{j} - (3xz^2 + 1)\hat{k}$ is conservative and find a scalar function ϕ such that $\vec{F} = -\vec{\nabla}\phi$.

b) Consider the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$, Find the unit normal $\hat{n}$ for this surface. Evaluate $(\vec{\nabla} \times \vec{v})\hat{n}$ with $\vec{v} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$ and hence find $\int\int (\vec{\nabla} \times \vec{v}).\hat{n}\, ds$ over the hemisphere. [4+6]

4. a) State the Dirichlet conditions on a Function such that it can be expanded in Fouries series. Can $f(x) = \tan x$ be expanded in a Fourier series? Give reason.

b) Find the Fourier series expansion for the function $f(x) = \begin{cases} 0, -\pi < x < 0 \\ x, \ 0 < x < \pi \end{cases}$

a) Check the convergence of <u>any two</u> of the following series :

i) $\sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}}}{(n+1)^{\frac{1}{2}}}$ ii) $\sum_{n=1}^{\infty} \frac{10^n}{(n!)^2}$ iii) $\sum_{n=1}^{\infty} \frac{(-1)^n (2n+1)}{n}$ iv) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{2}}}$ [2+4+4]

5. a) Considering the Gaussian distribution function of the random variable x,

$f(x) = \frac{1}{\sqrt{2\pi\lambda}} \exp\left(-\frac{(x-\lambda)^2}{2\lambda}\right)$, show that the mean $\mu = \lambda$, and the variance $\sigma^2 = \lambda$

b) Let $\vec{a}_1 = (-1,1,1),\ \vec{a}_2 = (1,-1,1), \vec{a}_3 = (1,1,-1)$

i) Show that $\{\vec{a}_1, \vec{a}_2, \vec{a}_3\}$ form a basis.

ii) Obtain a reciprocal basis $\{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$, with $\vec{a}_i.\vec{b}_j = \delta_{ij}(i, j = 1, 2, 3)$

iii) Express the vector $\vec{A}$ = (2,1,3) in terms of the reciprocal basis. [4+6]

6. a) Discuss the behaviour of the solution of the differential equation,
$y''(x) + 2\lambda y'(x) + k^2 y(x) = 0, (k > 0)$ for (i) $\lambda = 0$, (ii) $\lambda < k$, (iii) $\lambda = k$

b) Use Frobenius' method to obtain a general solution of <u>any one</u> of the following differential equations :

i) $4xy'' + 2y' + y = 0$

ii) $2x^2 y'' + 3xy' + (x-1)y = 0$ [5+5]

7. a) Assuming $e^{2tx-t^2} = \sum_{n=0}^{\infty} \frac{t^n}{n!} H_n(x)$ prove that $2nH_{n-1}(x) = H'_n(x)$

b) Assuming the solution of Laplace's equation $\nabla^2 v = 0$ at any point (r, θ, ϕ), find the potential at the point when a conducting sphere of radius 'a' is placed in a uniform electric field of intensity E, along the z-axis. The boundary conditions are

i) V = 0, at r = a, with origin at centre of sphere,

ii) V = –EZ, when $r \to \infty$ [4+6]

<u>Group – B</u>

Answer <u>any two</u> questions from the following : [10×2 = 20]

8. a) Compare the action of Huygens' and Ramsden's eyepiece in reference to spherical aberration, Chromatic aberration and actual measurement of the objective image.

b) What do you mean by the terms 'field of view', 'entrance and exit pupils' of an optical instrument? [6+4]

9. a) Derive Helmholtz Lagrange relation between linear magnification and angular magnification of an optical system. Show that when the media on the two sides of a lens are same, the nodal points coincide with the principal points.

b) Define dispersive power. Is it a constant for a given material?

c) It is desired to make a converging achromatic doublet of focal length 30 cm by using two lenses of materials A and B whose dispersive powers are in the ratio 1:2. Find the focal length of each lens. [5+2+3]

10. a) Write down the translation matrix and refraction matrix of an optical system for paraxial rays. Hence deduce the system matrix for a thin lens.

b) Find the positions of cardinal points and equivalent focal length for a combination of two convex lenses of focal lengths 20cm and 10cm situated at a distance of 10cm apart in air. Draw the ray diagram. (Use any method) [5+5]

Answer <u>any one</u> question from the following : [5×1 = 5]

11. a) Define cardinal points of an optical system.

b) Discuss any two methods of removing spherical aberration. [2+3]

12. a) State Fermat's principle of extremum path. Establish the principle of reversibility of light rays from Fermat's principle.

b) What is aplanatic surface? Give an example. [3+2]